The solution of the system (4.7) reduces to the solution of one nonlinear ordinary differential equation with a singularity (of the "saddle" type) [10]. Hence, obtaining the final expressions for the nonstationary relaxing stream parameters flowing perpendicularly to the wing set up turns out to be considerably more tedious than in case 'b'.

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AN EXACT SOLUTION FOR THE INTERACTION OF A SUPERSONIC WEDGE WITH THE BOUNDARY BETWEEN TWO GASES

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It is fairly complicated to examine the interaction of a moving body with inhomogeneities (shock waves or contact discontinuities) in a gas flow. The problem is a nonlinear nonstationary one, in which there is a series of interactions between the shock waves, contact discontinuities, and expansion waves. Therefore, only the linear formulation has been used in analytic solution in [1-3].

In the general case, the solution can be found only numerically [4-6]. Exact solutions can be found in certain cases. For example, in [7, 8] there are exact solutions for the flow of an incident shock wave around a moving wedge.

Here we derive a class of exact solutions for the interaction of a wedge moving with a supersonic velocity in an ideal gas with the boundary between two gases. The medium is considered nonviscous.

1. We consider a wedge with a semivertex angle θ (Fig. 1) moving with a supersonic velocity q_0 in a medium where the pressure, density, and adiabatic parameter are correspondingly $p_0 = 1$, $\rho_0 = 1$, γ_0; there is incident on the wedge at some angle β to the axis of motion a contact discontinuity DBF, where DB is part of the surface of the discontinuity that has not yet interacted, BF is the new surface of the discontinuity, ABC is the head shock wave, BE is the shock wave reflected from the surface of the contact discontinuity, and φ is the angle formed by the head wave. We examined the flow picture on the upper surface of the wedge subject to the condition that the shock wave BE is reflected from the contact discontinuity. The case with a negative-pressure wave is not considered.

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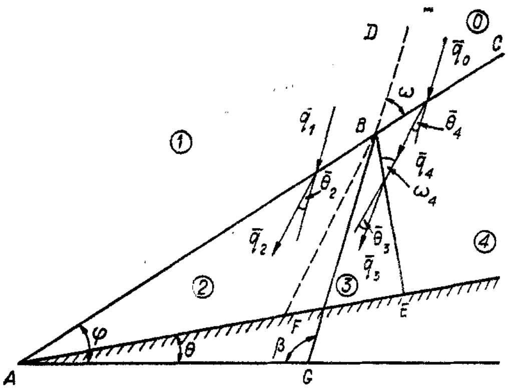

Fig. 1

In the general case, the interaction picture is much more complicated than that in Fig. 1. When the shock wave BC falls on the contact discontinuity BD, the reflected shock wave BE in turn is reflected from the surface of the wedge and interacts with the other discontinuities. The head wave ABC should be refracted at point B. The contact discontinuity BF is reflected from the surface of the body as a compression or expansion wave. If on the other hand it is required that shock wave BE is perpendicular to the surface of the wedge, the velocities will be equal in regions 2 and 3, and wave ABC is not refracted, in which case we get the simple flow picture shown in Fig. 1. Then the velocity should be supersonic in all the regions shown with respect to the coordinate system related to point B.

We derive the relationship that must be imposed on the flow parameters in regions 0 and 1 to get the flow picture shown in Fig. 1.

In the region of point B, we have a pattern of overall regular refraction of shock wave ABC at the discontinuity DBF. A difference from the theory of ordinary regular shock-wave refraction at the boundary of two media [9] is that here there is a velocity discontinuity at the first contact discontinuity. This introduces an additional arbitrary parameter into the refraction problem. One usually employs the following parameters to describe the physical state in two regions separated by a contact-discontinuity surface: the ratios of the specific heats γ_0 and γ_1, the speeds of sound a_0 and a_1, and the pressures p_0 and p_1. At equilibrium, $p_0 = p_1$.

Here we incorporate the velocity discontinuity at the interface and use the velocities q_0 and q_1 also to describe the states of the gas in the two regions. We formulate the problem in a coordinate system linked to the triple point B and moving with a constant velocity q. The components of this along the x and y axes are

$$q_x = \frac{q_0 \sin\beta \cdot \cos\varphi}{\sin\omega}, \quad q_y = \frac{q_0 \sin\beta \cdot \sin\varphi}{\sin\omega}.$$

Here φ is the angle of the head shock wave, ω is the angle between the interface between the two gases and shock wave ABC, and β is the angle formed by the interface with the x axis, which coincides with the symmetry axis of the wedge.

In the subsequent expressions, the values of q and θ in regions 0–4 as calculated in the coordinate system linked to the mobile point B will differ from those calculated in the coordinate system related to the leading edge of the wedge.

The flow picture (Fig. 1) implies that to solve the problem we need to find the components of the velocities, the pressures, and the densities in regions 2–4. There are 12 relationships on the shock waves AB, BC, and BE. Also, the problem contains 6 unknown quantities ω, β, ω_4, u_1, v_1, ρ_1, for which there are additional defining conditions. Here ω_4 is the angle formed by wave BE and the direction of the gas velocity vector in region 4, while u_1 and v_1 are the components of q_1 along the x and y axes in the coordinate system linked to the leading edge of the wedge.

Geometrical considerations show that β is expressed as

$$\beta = 180° - \varphi - \omega. \tag{1.1}$$

TABLE 1

$q_0=2{,}52$;	$M_0=2{,}4$;	$\gamma_0=1{,}1$;	$\gamma_1=1{,}4$;	$\theta=10°$;	$\varphi=31{,}8°$
$p_1=1$		$\rho_1=1{,}38$	$q_1=2{,}54$	$\theta_1=-1{,}45°$	
$p_2=1{,}72$		$\rho_2=2{,}02$	$q_2=2{,}36$	$\theta_2=10°$	
$p_3=1{,}72$		$\rho_3=1{,}63$	$q_3=2{,}36$	$\theta_3=10°$	
$p_4=1{,}62$		$\rho_4=1{,}55$	$q_4=2{,}31$	$\theta_4=10°$	

$\omega=40{,}9°$, $\beta=107{,}3°$

The condition that wave BE is perpendicular to the surface of the wedge gives us as follows for ω_4:

$$\omega_4 = 90° + \bar{\theta}_4 - \varphi - \omega, \tag{1.2}$$

where $\bar{\theta}_4$ is the deviation angle of the flow behind wave BC.

Apart from these two geometrical relationships, four further conditions related to the geometrical and gasdynamic characteristics of the flow should be obeyed at the contact surfaces BD and BF.

Firstly, the velocities q_0 and q_1 cannot take arbitrary values, and on BD the normal components of these should be equal. In the coordinate system linked to the tip of the wedge, this is equivalent to the condition

$$q_1\sin(\beta-\theta_1) = q_0\sin\beta, \tag{1.3}$$

where $\theta_1 = \operatorname{arctg} u_1/v_1$.

Secondly, the condition for equality of the pressure should be obeyed at the contact surface:

$$p_2 = p_3. \tag{1.4}$$

Thirdly, the condition for parallelism of the flows should be obeyed on this:

$$\bar{\theta}_4 - \bar{\theta}_3 = \bar{\theta}_2, \tag{1.5}$$

where $\bar{\theta}_2$ and $\bar{\theta}_3$ are the flow deviation angles behind the waves AB and BE.

Fourthly, the velocities in regions 2 and 3 should be equal:

$$\bar{q}_2 = \bar{q}_3. \tag{1.6}$$

Equations (1.1)-(1.6) together with the relationships describing the transition to the shock waves AB, BC, and BE constitute a complete mathematical solution for the interaction of a moving wedge with a boundary between two media, i.e., the configuration of the interacting waves and contact surfaces is completely defined for given parameters γ_0, γ_1, q_0, θ, p_0, ρ_0, and the density, velocity, and direction of the velocity in region 1 are also determined.

2. Consider the solution to (1.1)-(1.6). Equation (1.4) is equivalent to

$$(1+\mu_1)M_1^2\sin^2\omega - \mu_1 = \xi_4\{(1+\mu_0)M_4^2\sin^2\omega_4 - \mu_0\} = \xi_2, \tag{2.1}$$

where ξ and η (with the corresponding subscripts) denote the intensity of the corresponding shock wave and the density ratio at it,

$$\mu_0 = \frac{\gamma_0-1}{\gamma_0+1};\ \mu_1 = \frac{\gamma_1-1}{\gamma_1+1};\ M_4^2 = \frac{\bar{q}_4^2\eta_4}{\gamma_0\xi};$$
$$\eta_4 = \frac{\rho_4}{\rho_1} = \frac{\xi_4+\mu_0}{\xi_4\mu_0+1};\quad \bar{q}_4^2 = \bar{q}_0^2\sin^2\omega\left(\operatorname{ctg}^2\omega + \frac{1}{\eta_4^2}\right).$$

In (1.1)-(1.6) and (2.1) and subsequently, the velocities and angles are taken in the coordinate system related to point B if no special mention is made.

Equation (1.5) is transformed to

$$\frac{\operatorname{tg}\omega(\eta_4-\eta_2)}{\eta_4\eta_2+\operatorname{tg}^2\omega} = \frac{\operatorname{tg}\omega_4(\eta_3-1)}{\eta_3+\operatorname{tg}^2\omega_4}. \tag{2.2}$$

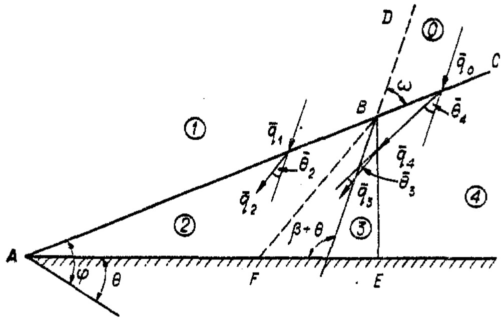

Fig. 2

The condition for equality of the velocities in regions 2 and 3 gives an additional relation between the velocities in regions 0 and 1:

$$\bar{q}_1^2 = \bar{q}_0^2\left(\cos^2\omega + \sin^2\omega/\eta_4^2\right)\left(\cos^2\omega_4 + \sin^2\omega_4/\eta_3^2\right)/\left(\cos^2\omega + \sin^2\omega/\eta_2^2\right). \tag{2.3}$$

Using (1.2) and (2.1), we reduce (2.2) to a transcendental equation for the unknown ω, whose roots represent a complete solution. The quantities ρ_1 and $\bar{q}_1$ are found from (2.3) and the left part of (2.1). To find the direction and magnitude of q_1 in the coordinate system linked to the tip of the wedge, we use condition (1.3).

The roots of (2.2) were found numerically by a computer. We considered a wide range in the initial parameters: $1.1 \leqslant \gamma_0;\ \gamma_1 \leqslant 2.1;\ 1.4 \leqslant q_0 \leqslant 4;\ 10° \leqslant \theta \leqslant 30°$. The steps in γ_0 and γ_1 were 0.1, while q_0 was about 0.5 and the wedge angle was about 10°. In the calculations it was found that this exact solution applies only for $\gamma_1 > \gamma_0$. In fact, the right side in (2.2) is always positive, and therefore we should have

$$\eta_4 - \eta_2 = \frac{\xi_4 + \mu_0}{\xi_4\mu_0 + 1} - \frac{\xi_4\xi_3 + \mu_1}{\xi_4\xi_3\mu_1 + 1} > 0.$$

This is equivalent to

$$(\mu_1 - \mu_0)\left(\xi_4^2\xi_3 - 1\right) + \xi_4(1 - \mu_0\mu_1)(1 - \xi_3) > 0. \tag{2.4}$$

The second term in (2.4) is always less than zero, and therefore if this is to be obeyed the first term must always be greater than zero, which is possible only for $\mu_1 > \mu_0$ or $\gamma_1 > \gamma_0$.

Figures 1 and 2 show the general picture for the interaction of the moving wedge with the boundary between the two media. Note that for convenience in representation, the entire flow picture in Fig. 2 has been turned through 30° clockwise. The values of the gas-dynamic quantities for these two forms are given in Tables 1 and 2, where the first line gives the initial parameters and the velocities have been calculated in the coordinate system linked to the wedge tip.

The arrows in Figs. 1 and 2 show the directions of the velocity vectors in regions 0-4 in the mobile coordinate system. The deviation angles for the velocity vectors in Fig. 1 are $\bar{\theta}_2 = 10.3°$, $\bar{\theta}_3 = 1.4°$, $\bar{\theta}_4 = 11.7°$. With the stronger shock wave in Fig. 2, these deviations are more substantial: $\bar{\theta}_2 = 20.4°$, $\bar{\theta}_3 = 8.8°$, $\bar{\theta}_4 = 29.2°$. The calculations show that the intensity of wave BE is fairly small for a thin wedge, but it increases with γ_1, q_0, θ; β varies over the range 75-115° in the above range for the initial parameters.

Figures 3 and 4 show the behavior of p_3/p_4 as a function of the Mach number of the incident flow and γ_1. Curves 1-3 relate to wedge thickness $\theta = 10; 20; 30°$. In Fig. 3, the fixed parameters are $\gamma_0 = 1.1$ and $\gamma_1 = 1.9$, while in Fig. 4 they are $\gamma_0 = 1.1$ and $M_0 = 2.4$.

We see that when a strong wave falls on the interface between two gases, the intensity of wave BE increases by a factor 1.75.

TABLE 2

$q_0=2,52$;	$M_0=2,4$;	$\gamma_0=1,1$;	$\gamma_1=1,9$; $\theta=30°$; $\varphi=51,8°$
$p_1 = 1$	$\rho_1 = 3.15$	$q_1 = 2.47$	$\theta_1 = -11.7°$
$p_2 = 5.19$	$\rho_2 = 6.62$	$q_2 = 2,03$	$\theta_2 = 30°$
$p_3 = 5.19$	$\rho_3 = 4.33$	$q_3 = 2,03$	$\theta_3 = 30°$
$p_4 = 3,68$	$\rho_4 = 3.18$	$q_4 = 1,68$	$\theta_4 = 30°$

$\omega = 49.6°$, $\beta = 79.6°$

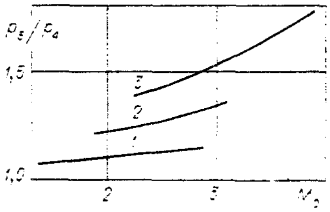

Fig. 3

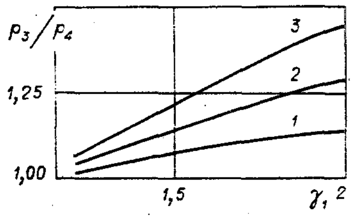

Fig. 4

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